



**Amherst County
Public Schools**

6th Grade Mathematics – Curriculum

LESS THAN



GREATER THAN

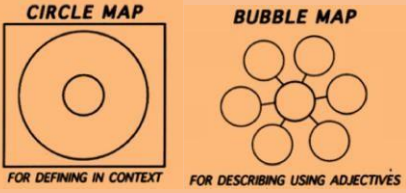


LESS THAN OR EQUAL TO



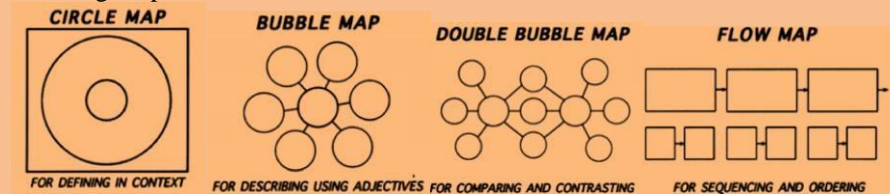
GREATER THAN OR EQUAL TO



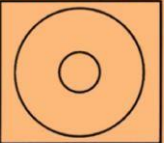

SOL	6.1 Resources
Standard	The student will represent relationships between quantities using ratios, and will use appropriate notations, such as $\frac{a}{b}$, a to b, and $a:b$.
Days	5 days
Key Vocabulary	ratio relationship quantity fraction equivalent compare comparison symbolic simplest form related whole unit rate numerator denominator rate
Essential Questions	How can you use a ratio to compare two quantities?
Foundational Objectives	No prior standards.
Succeeding Objectives	7.2 The student will solve practical problems involving operations with rational numbers. 7.3 The student will solve single-step and multistep practical problems, using proportional reasoning. 7.10 The student will a) determine the slope, m , as rate of change in a proportional relationship between two quantities .
Thinking Map	
 <p>The image shows two thinking map diagrams. On the left is a 'CIRCLE MAP' consisting of a large outer circle and a smaller inner circle, with the text 'FOR DEFINING IN CONTEXT' below it. On the right is a 'BUBBLE MAP' consisting of a central circle surrounded by seven smaller circles, with the text 'FOR DESCRIBING USING ADJECTIVES' below it.</p>	

SOL	6.2 Resources		
Standard	The student will a) represent and determine equivalencies among fractions, mixed numbers, decimals, and percents;* and b) compare and order positive rational numbers.* *On the state assessment, items measuring this objective are assessed without the use of a calculator		
Days	10 days		
Key Vocabulary	Ascending Compare Comparison Decimal Denominator Descending Equivalent Fraction	Improper Fraction Mixed number Model(Pictorial Representation) Numerator Order Percent	Proper Fraction Ratio Rational Numbers Repeating Decimal Simplify Terminating Decimal
Essential Questions	How can a model help show the relationship between equivalent fractions, decimals, and percents? When shopping, how can knowing the equivalent form of a fraction, decimal and percents help when calculating a price? Traveling? When is it best to use a fraction? When is it best to use a decimal? When is it best to use a percent? How can you demonstrate that one percent is greater than another? How can I determine whether a number is equivalent to, greater than, less than another number? How is it possible to compare/order numbers that are represented in different formats?		
Foundational Objectives	5.2 The student will a) represent and identify equivalencies among fractions and decimals, with and without models; and b) compare and order fractions, mixed numbers, and/or decimals in a given set, from least to greatest and greatest to least.		
Succeeding Objectives	7.1 The student will a) investigate and describe the concept of negative exponents for powers of ten; b) compare and order numbers greater than zero written in scientific notation;* c) compare and order rational numbers;*		

Thinking Map



SOL	6.3 Resources	
Standard	The student will a) identify and represent integers; b) compare and order integers; and c) identify and describe absolute value of integers	
Days	Approx. 5 days	
Key Vocabulary	Whole number Integers Set positive negative opposite absolute value zero pair	compare order greater than greater than or equal to less than less than or equal to
Essential Questions	How does a number line help to compare two integers? Are negative integers always less than positive integers? Justify your answer. Why do we use the absolute value of a number when talking about distance? How does the opposite of n differ from the absolute value of n ?	
Foundational Objectives	none	
Succeeding Objectives	7.1 The student will c) compare and order rational numbers; e) Identify and describe absolute value of rational numbers.	
Thinking Map	<p> CIRCLE MAP FOR DEFINING IN CONTEXT </p> <p> BUBBLE MAP FOR DESCRIBING USING ADJECTIVES </p> <p> DOUBLE BUBBLE MAP FOR COMPARING AND CONTRASTING </p>	

SOL	6.4 Resources
Standard	The student will recognize and represent patterns with whole number exponents and perfect squares.
Days	5 days
Key Vocabulary	<ul style="list-style-type: none"> Base Exponent Exponential Notation Factor Integers Perfect Square Power Square Root Whole Number Pattern Place value
Essential Questions	<ul style="list-style-type: none"> How can patterns be used to make predictions? How does a perfect square relate to a geometric square?
Foundational Objectives	No prior standards.
Succeeding Objectives	7.1 The student will d) determine square roots of perfect squares
Thinking Map	
<div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> <p>CIRCLE MAP</p>  <p>FOR DEFINING IN CONTEXT</p> </div> <div style="text-align: center;"> <p>BUBBLE MAP</p>  <p>FOR DESCRIBING USING ADJECTIVES</p> </div> </div>	

SOL	6.5 Resources		
Standard	<p>The student will</p> <p>a) multiply and divide fractions and mixed numbers;*</p> <p>b) solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions and mixed numbers; and</p> <p>c) solve multistep practical problems involving addition, subtraction, multiplication, and division of decimals.</p> <p>*On the state assessment, items measuring this objective are assessed without the use of a calculator.</p>		
Days	15 days		
Key Vocabulary	fraction mixed number improper fraction equivalent fraction simplest form numerator	denominator decimal factor product quotient divisor	dividend common factor estimate approximate model regrouping
Essential Questions	Where in a fractions multiplication model do you find the factors? Where is the product? Why are common denominators needed sometimes and not other times? How do I make a reasonable estimate? Why is estimating important when solving real world problems with fractions and decimals? How do I know when a result is reasonable? How can you tell which operations are required to solve real world problems? How do I decide what strategy will work best in a given problem situation? How can you use (apply) decimal operations in real life?		
Foundational Objectives	5.5 The student will a) estimate and determine the product and quotient of two numbers involving decimals; and b) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication of decimals, and create and solve single-step practical problems involving division of decimals. 5.6 The student will a) solve single-step and multistep practical problems involving addition and subtraction with fractions and mixed numbers; and b) solve single-step practical problems involving multiplication of a whole number, limited to 12 or less, and a proper fraction, with models.		
Succeeding Objectives	7.2 The student will solve practical problems involving operations with rational numbers. 7.3 The student will solve single-step and multistep practical problems, using proportional reasoning.		
Thinking Map	<p>CIRCLE MAP FOR DEFINING IN CONTEXT</p> <p>TREE MAP FOR CLASSIFYING AND GROUPING</p> <p>BUBBLE MAP FOR DESCRIBING USING ADJECTIVES</p> <p>DOUBLE BUBBLE MAP FOR COMPARING AND CONTRASTING</p> <p>FLOW MAP FOR SEQUENCING AND ORDERING</p>		

SOL	6.6 Resources		
Standard	<p>The student will</p> <p>a) add, subtract, multiply, and divide integers;*</p> <p>b) solve practical problems involving operations with integers; and</p> <p>c) simplify numerical expressions involving integers.*</p> <p>*On the state assessment, items measuring this objective are assessed without the use of a calculator.</p>		
Days	Approx. 20 days		
Key Vocabulary	Integer set whole number opposite sum difference	product quotient positive negative grouping symbols	parentheses division bar exponents power simplify numerical expression
Essential Questions	<p>How does the knowledge of zero pairs help when modeling operations with integers?</p> <p>Under what circumstances will the sum or difference of integers result in a negative solution?</p> <p>Under what circumstances will the product or quotient result in a negative solution?</p> <p>What strategies are most useful in helping develop algorithms for adding, subtracting, multiplying, and dividing positive and negative numbers?</p> <p>Will addition of integers ever result in a sum smaller than one or smaller than both of its addends?</p> <p>Will subtraction of integers ever yield a difference greater than the minuend and/or subtrahend?</p> <p>When will the sum of two integers be positive? Negative? Or zero?</p> <p>What is a real world situation in which you have to add, subtract, multiply or divide both positive and negative integers?</p> <p>How do you use integer operations to balance a checkbook or budget?</p> <p>Why do we need an order of operations?</p> <p>Why is multiplication and division performed from left to right?</p> <p>Why is addition and subtraction performed from left to right?</p> <p>If there are two different addition problems in an expression, is it ok to do the second one first? Explain using the properties of real numbers.</p>		
Foundational Objectives	5.7 The student will simplify whole number numerical expressions using the order of operations.		
Succeeding Objectives	7.2 The student will solve practical problems involving operations with rational numbers.		

Thinking Map

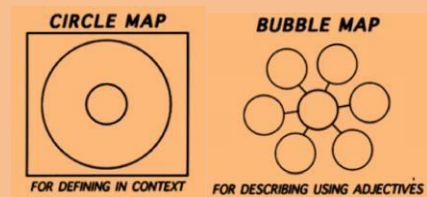


SOL	6.7 Resources	
Standard	The student will a) derive π (pi); b) solve problems, including practical problems, involving circumference and area of a circle; and c) solve problems, including practical problems, involving area and perimeter of triangles and rectangles.	
Days	10 days	
Key Vocabulary	circle pi approximation radius circumference diameter area triangle	rectangle base height length width perimeter
Essential Questions	<ul style="list-style-type: none"> • What relationships exist among the radius, diameter, and circumference of a circle? • In what type of situations would you need to find the area of a circle? • How can patterns be used to determine formulas for area and perimeter? • How could you find the area of a square if you knew the perimeter? • How is the area formula for a triangle similar to the area formula of a rectangle? 	
Foundational Objectives	5.8 The student will a) solve practical problems that involve perimeter, area, and volume in standard units of measure; and b) differentiate among perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation. 5.10 The student will identify and describe the diameter, radius, chord, and circumference of a circle.	
Succeeding Objectives	7.4 The student will a) describe and determine the volume and surface area of rectangular prisms and cylinders; and b) solve problems, including practical problems, involving the volume and surface area of rectangular prisms and cylinders.	
Thinking Map	<p>CIRCLE MAP FOR DEFINING IN CONTEXT</p> <p>TREE MAP FOR CLASSIFYING AND GROUPING</p> <p>BUBBLE MAP FOR DESCRIBING USING ADJECTIVES</p>	

SOL	6.8 Resources
Standard	<p>The student will</p> <p>a) identify the components of the coordinate plane; and</p> <p>b) identify the coordinates of a point and graph ordered pairs in a coordinate plane.</p>
Days	5 days
Key Vocabulary	coordinate plane axis quadrant ordered pair origin distance horizontal vertical coordinate x-coordinate y-coordinate
Essential Questions	How are the axes of a coordinate plane related to a number line? On a coordinate plane, how are points on a horizontal line related to each other? On a coordinate plane, how are points on a vertical line related to each other? How do you determine the distance a point is from an axes?
Foundational Objectives	4.14 The student will a) collect, organize, and represent data in bar graphs and line graphs
Succeeding Objectives	7.10 The student will b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in $y = mx$ form where m represents the slope as rate of change; d) graph a line representing an additive relationship between two quantities given the y-intercept and an ordered pair, or given the equation in the form $y = x + b$, where b represents the y-intercept; and e) make connections between
Thinking Map	
<p>The Thinking Map section displays three types of maps:</p> <ul style="list-style-type: none"> CIRCLE MAP: A diagram with a large outer circle and a smaller inner circle. Below it is the text "FOR DEFINING IN CONTEXT". BUBBLE MAP: A diagram with a central circle connected to six surrounding circles. Below it is the text "FOR DESCRIBING USING ADJECTIVES". FLOW MAP: A diagram showing a sequence of boxes. The top row has three boxes connected by arrows from left to right. The bottom row has four boxes connected by arrows from left to right. Below it is the text "FOR SEQUENCING AND ORDERING". 	

SOL	6.9 Resources
Standard	The student will determine congruence of segments, angles, and polygons.
Days	5 days
Key Vocabulary	congruent segments angles Regular polygons symmetry sides line of symmetry noncongruent
Essential Questions	<ul style="list-style-type: none"> • How does the line create 2 congruent parts when certain shapes are divided with a line of symmetry? • How do you determine if two figures are congruent?
Foundational Objectives	5. 12 The student will classify and measure right, acute, obtuse, and straight angles.
Succeeding Objectives	7.5 The student will solve problems, including practical problems, involving the relationship between corresponding sides and corresponding angles of similar quadrilaterals and triangles.

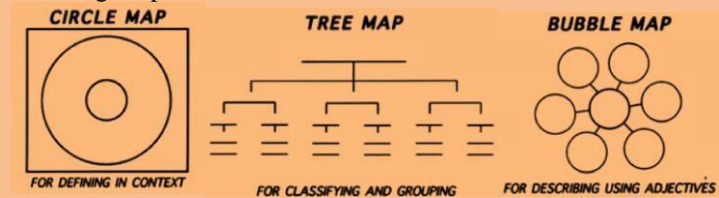
Thinking Map

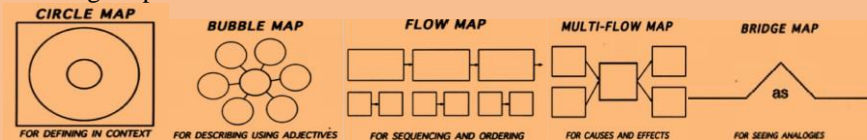


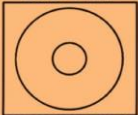
SOL	6.10 Resources	
Standard	<p>The student will</p> <p>a) represent data in a circle graph;</p> <p>b) make observations and inferences about data represented in a circle graph; and</p> <p>c) compare circle graphs with the same data represented in bar graphs, pictographs, and line plots.</p>	
Days	Approx. 10 days	
Key Vocabulary	circle graph data observation pictograph line plot frequency	scale trends predictions inference bar graph
Essential Questions	<ul style="list-style-type: none"> • How does a circle graph help me organize my data? • Is it possible to manipulate data to change the way the data is perceived? • How does the type of data influence the choice of graph? 	
Foundational Objectives	5.16 The student, given a practical problem, will a) represent data in line plots and stem-and-leaf plots; b) interpret data represented in line plots and stem-and-leaf plots; and c) compare data represented in a line plot with the same data represented in a stem-and leaf plot.	
Succeeding Objectives	7.9 The student, given data in a practical situation, will a) represent data in a histogram; b) make observations and inferences about data represented in a histogram; and c) compare histograms with the same data represented in stem-and-leaf plots, line plots, and circle graphs.	
Thinking Map		

SOL	6.11 Resources
Standard	<p>The student will</p> <p>a) represent the mean of a data set graphically as the balance point; and</p> <p>b) determine the effect on measures of center when a single value of a data set is added, removed, or changed.</p>
Days	Approx. 5 days
Key Vocabulary	Mean balance point line plot median mode average measures of center Data set
Essential Questions	<ul style="list-style-type: none"> • How the mean is graphically represented as a balance point? • In which situations would the mean, median, or mode be the best representation of the data? • What effect does adding or removing one piece of data have on the mean? • What effect does adding or removing one piece of data have on the mode? • What effect does adding or removing one piece of data have on the median?
Foundational Objectives	5.17 The student, given a practical context, will a) describe mean, median, and mode as measures of center; b) describe mean as fair share; c) describe the range of a set of data as a measure of spread; and d) determine the mean, median, mode, and range of a set of data.
Succeeding Objectives	7.9 The student, given data in a practical situation, will b) make observations and inferences about data represented in a histogram.

Thinking Map



SOL	6.12 Resources			
Standard	<p>The student will</p> <p>a) represent a proportional relationship between two quantities, including those arising from practical situations;</p> <p>b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;</p> <p>c) determine whether a proportional relationship exists between two quantities; and</p> <p>d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.</p>			
Days	Approx. 15 days			
Key Vocabulary	ratio unit rate ratio table multiplicative relationships	equivalent equivalent ratio proportion proportional relationship	input/output tables double number line graphs verbal descriptions	ordered pairs coordinate plane multiple representations constant
Essential Questions	<ul style="list-style-type: none"> • What is the relationship between a ratio and a proportion? • How does a proportion compare to two equivalent ratios? • How would you use a table to show a proportional relationship? • What is a situation that demonstrates a proportional relationship? • What is a situation that does not demonstrate a proportional relationship? • When would a unit rate be useful to solve a problem? • How do you interpret a unit rate? • What are the similarities between simplifying a fraction and showing proportional relationship? • What is a proportional relationship? • How is a proportional relationship, when written in words, shown in a ratio table and a graph? 			
Foundational Objectives	No prior standards			
Succeeding Objectives	<p>7.10 The student will a) determine the slope, m, as rate of change in a proportional relationship between two quantities and write an equation in the form $y = mx$ to represent the relationship; b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in $y = mx$ form where m represents the slope as rate of change; c) determine the y-intercept, b, in an additive relationship between two quantities and write an equation in the form $y = x + b$ to represent the relationship; d) graph a line representing an additive relationship between two quantities given the y-intercept and an ordered pair, or given the equation in the form $y = x + b$, where b represents the y-intercept; and e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.</p>			
<p>Thinking Map</p>  <p>CIRCLE MAP FOR DEFINING IN CONTEXT</p> <p>BUBBLE MAP FOR DESCRIBING USING ADJECTIVES</p> <p>FLOW MAP FOR SEQUENCING AND ORDERING</p> <p>MULTI-FLOW MAP FOR CAUSES AND EFFECTS</p> <p>BRIDGE MAP FOR SEEING ANALOGIES</p>				

SOL	6.13 Resources		
Standard	The student will solve one-step linear equations in one variable, including practical problems that require the solution of a one-step linear equation in one variable.		
Days	Approx.15 days		
Key Vocabulary	equation variable expression term coefficient variable expression algebraic expression solution	commutative property of addition commutative property of multiplication identity property of addition identity property of multiplication inverses inverse property of addition	inverse property of multiplication multiplicative property of zero addition property of equality subtraction property of equality multiplication property of equality division property of equality substitution property
Essential Questions	<ul style="list-style-type: none"> • How can a model be used to represent an equation? • How can a model be used to solve an equation? • How can you check to see if your solution is correct? • How can a practical situation be represented by a multi-step equation? • How can understanding properties help when solving an equation? • How does the knowledge of zero pairs help when solving equations? • How can you represent an equation in a model? Picture? • How do the properties contribute to algebraic understanding? • What strategies can be used to solve for unknowns in an algebraic expression? • How are the four basic operations related to one another? • What strategies can you use to determine if your solution is correct? • How can you use words to represent an equation or expression? 		
Foundational Objectives	5.19 The student will a) investigate and describe the concept of variable; b) write an equation to represent a given mathematical relationship, using a variable; c) use an expression with a variable to represent a given verbal expression involving one operation; and d) create a problem situation based on a given equation, using a single variable and one operation.		
Succeeding Objectives	7.12 The student will solve two-step linear equations in one variable, including practical problems that require the solution of a two-step linear equation in one variable.		
Thinking Map	 <p>CIRCLE MAP FOR DEFINING IN CONTEXT</p>		

SOL	6.14 Resources		
Standard	The student will a) represent a practical situation with a linear inequality in one variable; and b) solve one-step linear inequalities in one variable, involving addition or subtraction, and graph the solution on a number line.		
Days	Approx. 10 days		
Key Vocabulary	inequality variable greater than less than greater than or equal to less than or equal to open circle closed circle	solution set commutative property of addition commutative property of multiplication identity property of addition identity property of multiplication inverses inverse property of addition	inverse property of multiplication multiplicative property of zero addition property of inequality subtraction property of inequality multiplication property of inequality division property of inequality substitution property
Essential Questions	<ul style="list-style-type: none"> • How can you compare and contrast the process for solving an equation and inequality? • How can you represent an inequality in a model? • How can you compare and contrast the solutions to an equation and inequalities? • When are inequalities used in the real world? • How can we tell if a solution should be graphed with an open or closed circle? • How can we tell if a number is a solution from looking at the graph? Equation? • How do the properties contribute to solving inequalities? 		
Foundational Objectives	No prior standards.		
Succeeding Objectives	7.13 The student will solve one- and two-step linear inequalities in one variable, including practical problems, involving addition, subtraction, multiplication, and division, and graph the solution on a number line.		
Thinking Map	<p> CIRCLE MAP FOR DEFINING IN CONTEXT </p> <p> TREE MAP FOR CLASSIFYING AND GROUPING </p> <p> BUBBLE MAP FOR DESCRIBING USING ADJECTIVES </p>		

Virginia 2016 Mathematics Standards of Learning Curriculum Framework
Introduction

The 2016 *Mathematics Standards of Learning Curriculum Framework*, a companion document to the 2016 *Mathematics Standards of Learning*, amplifies the *Mathematics Standards of Learning* and further defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and *Curriculum Framework* are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and *Curriculum Framework* into a broader, locally designed curriculum. The *Curriculum Framework* delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students.

The *Curriculum Framework* also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the *Curriculum Framework*.

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the *Curriculum Framework* facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The *Curriculum Framework* is divided into two columns: Understanding the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard

This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction. The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s).

Essential Knowledge and Skills

This section provides a detailed expansion of the mathematics knowledge and skills that each student should know and be able to demonstrate. This is not meant to be an exhaustive list of student expectations.

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and to see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, “... the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations.” State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade three state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades four through seven, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must develop students’ conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of grade two and those for multiplication and division by the end of grade four. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. “Algebra readiness” describes the mastery of, and the ability to apply, the *Mathematics Standards of Learning*, including the Mathematical Process Goals for Students, for kindergarten through grade eight. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 *Mathematics Standards of Learning* form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics.

Equity

“Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.”

– National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students’ prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide

consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop problem-solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.

Mathematics instruction in grades six through eight continues to focus on the development of number sense, with emphasis on rational and real numbers. Rational numbers play a critical role in the development of proportional reasoning and advanced mathematical thinking. The study of rational numbers builds on the understanding of whole numbers, fractions, and decimals developed by students in the elementary grades. Proportional reasoning is the key to making connections to many middle school mathematics topics.

Students develop an understanding of integers and rational numbers using concrete, pictorial, and abstract representations. They learn how to use equivalent representations of fractions, decimals, and percents and recognize the advantages and disadvantages of each type of representation. Flexible thinking about rational number representations is encouraged when students solve problems.







Students develop an understanding of real numbers and the properties of operations on real numbers through experiences with rational and irrational numbers and apply the order of operations.

Students use a variety of concrete, pictorial, and abstract representations to develop proportional reasoning skills. Ratios and proportions are a major focus of mathematics learning in the middle grades.

6.1 The student will represent relationships between quantities using ratios, and will use appropriate notations, such as $\frac{a}{b}$, a to b , and $a:b$.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> • A ratio is a comparison of any two quantities. A ratio is used to represent relationships within a quantity and between quantities. Ratios are used in practical situations when there is a need to compare quantities. • In the elementary grades, students are taught that fractions represent a part-to-whole relationship. However, fractions may also express a measurement, an operator (multiplication), a quotient, or a ratio. Examples of fraction interpretations include: <ul style="list-style-type: none"> – Fractions as parts of wholes: $\frac{3}{4}$ represents three parts of a whole, where the whole is separated into four equal parts. – Fractions as measurement: the notation $\frac{3}{4}$ can be interpreted as three one-fourths of a unit. – Fractions as an operator: $\frac{3}{4}$ represents a multiplier of three-fourths of the original magnitude. – Fractions as a quotient: $\frac{3}{4}$ represents the result obtained when three is divided by four. – Fractions as a ratio: $\frac{3}{4}$ is a comparison of 3 of a quantity to the whole quantity of 4. • A ratio may be written using a colon ($a:b$), the word <i>to</i> (a to b), or fraction notation ($\frac{a}{b}$). • The order of the values in a ratio is directly related to the order in which the quantities are compared. <ul style="list-style-type: none"> – Example: In a certain class, there is a ratio of 3 girls to 4 boys (3:4). Another comparison that could represent the relationship between these quantities is the ratio of 4 boys to 3 girls (4:3). Both ratios give the same information about the number of girls and boys in the class, but they are distinct ratios. When you switch the order of comparison (girls to boys vs. boys to girls), there are different ratios being expressed. • Fractions may be used when determining equivalent ratios. <ul style="list-style-type: none"> – Example: The ratio of girls to boys in a class is 3:4, this can be interpreted as: number of girls = $\frac{3}{4}$ · number of boys. In a class with 16 boys, number of girls = $\frac{3}{4}$ · (16) = 12 girls. – Example: A similar comparison could compare the ratio of boys to girls in the class as being 4:3, which can be interpreted as: number of boys = $\frac{4}{3}$ · number of girls. 	<p>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</p> <ul style="list-style-type: none"> • Represent a relationship between two quantities using ratios. • Represent a relationship in words that makes a comparison by using the notations $\frac{a}{b}$, $a:b$, and a to b. • Create a relationship in words for a given ratio expressed symbolically.

6.1 The student will represent relationships between quantities using ratios, and will use appropriate notations, such as $\frac{a}{b}$, a to b , and $a:b$.


Understanding the Standard	Essential Knowledge and Skills														
<p>In a class with 12 girls, number of boys = $\frac{4}{3} \cdot (12) = 16$ boys.</p> <ul style="list-style-type: none"> A ratio can compare two real-world quantities (e.g., miles per gallon, unit rate, and circumference to diameter of a circle). Ratios may or may not be written in simplest form. A ratio can represent different comparisons within the same quantity or between different quantities. <table border="1" data-bbox="226 574 1094 1065"> <thead> <tr> <th colspan="2" style="background-color: #cccccc;"></th> </tr> </thead> <tbody> <tr> <td>part-to-whole (within the same quantity)</td> <td>compare part of a whole to the entire whole</td> </tr> <tr> <td>part-to-part (within the same quantity)</td> <td>compare part of a whole to another part of the same whole</td> </tr> <tr> <td>whole-to-whole (different quantities)</td> <td>compare all of one whole to all of another whole</td> </tr> <tr> <td>part-to-part (different quantities)</td> <td>compare part of one whole to part of another whole</td> </tr> </tbody> </table> <p>– Examples: Given Quantity A and Quantity B, the following comparisons could be expressed.</p> <table border="1" data-bbox="365 1263 961 1450"> <thead> <tr> <th>Quantity A:</th> <th>Quantity B:</th> </tr> </thead> <tbody> <tr> <td>  </td> <td>  </td> </tr> </tbody> </table>			part-to-whole (within the same quantity)	compare part of a whole to the entire whole	part-to-part (within the same quantity)	compare part of a whole to another part of the same whole	whole-to-whole (different quantities)	compare all of one whole to all of another whole	part-to-part (different quantities)	compare part of one whole to part of another whole	Quantity A:	Quantity B:			
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6.1 The student will represent relationships between quantities using ratios, and will use appropriate notations, such as $\frac{a}{b}$, a to b , and $a:b$.

Understanding the Standard			Essential Knowledge and Skills
	Ratio	Example	Ratio Notation(s)
	part-to-whole (within the same quantity)	compare the number of unfilled stars to the total number of stars in Quantity A	3:8; 3 to 8; or $\frac{3}{8}$
	part-to-part ¹ (within the same quantity)	compare the number of unfilled stars to the number of filled stars in Quantity A	3:5 or 3 to 5
	whole-to-whole ¹ (different quantities)	compare the number of stars in Quantity A to the number of stars in Quantity B	8:5 or 8 to 5
	part-to-part ¹ (different quantities)	compare the number of unfilled stars in Quantity A to the number of unfilled stars in Quantity B	3:2 or 3 to 2
<p>¹Part-to-part comparisons and whole-to-whole comparisons are ratios that are not typically represented in fraction notation except in certain contexts, such as determining whether two different ratios are equivalent.</p>			

- 6.2 The student will**
a) represent and determine equivalencies among fractions, mixed numbers, decimals, and percents;* and
b) compare and order positive rational numbers.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> • Fractions, decimals and percents can be used to represent part-to-whole ratios. <ul style="list-style-type: none"> – Example: The ratio of dogs to the total number of pets at a grooming salon is 5:8. This implies that 5 out of every 8 pets being groomed is a dog. This part-to-whole ratio could be represented as the fraction $\frac{5}{8}$ ($\frac{5}{8}$ of all pets are dogs), the decimal 0.625 (0.625 of the number of pets are dogs), or as the percent 62.5% (62.5% of the pets are dogs). • Fractions, decimals, and percents are three different ways to express the same number. Any number that can be written as a fraction can be expressed as a terminating or repeating decimal or a percent. • Equivalent relationships among fractions, decimals, and percents may be determined by using concrete materials and pictorial representations (e.g., fraction bars, base ten blocks, fraction circles, number lines, colored counters, cubes, decimal squares, shaded figures, shaded grids, or calculators). • <i>Percent</i> means “per 100” or how many “out of 100”; <i>percent</i> is another name for <i>hundredths</i>. • A number followed by a percent symbol (%) is equivalent to a fraction with that number as the numerator and with 100 as the denominator (e.g., $30\% = \frac{30}{100} = \frac{3}{10}$; $139\% = \frac{139}{100}$). • Percents can be expressed as decimals (e.g., $38\% = \frac{38}{100} = 0.38$; $139\% = \frac{139}{100} = 1.39$). • Some fractions can be rewritten as equivalent fractions with denominators of powers of 10, and can be represented as decimals or percents (e.g., $\frac{3}{5} = \frac{6}{10} = \frac{60}{100} = 0.60 = 60\%$). Fractions, decimals, and percents can be represented by using an area model, a set model, or a measurement model. For example, the fraction $\frac{1}{3}$ is shown below using each of the three models. <div style="text-align: center; margin: 10px 0;">  </div> <ul style="list-style-type: none"> • Percents are used to solve practical problems including sales, data description, and data comparison. • The set of rational numbers includes the set of all numbers that can be expressed as fractions in the form $\frac{a}{b}$ where a and b are integers and b does not equal zero. The decimal form of a rational number can be expressed as a terminating or repeating decimal. A few examples of positive rational numbers 	<p>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</p> <ul style="list-style-type: none"> • Represent ratios as fractions (proper or improper), mixed numbers, decimals, and/or percents. (a) • Determine the decimal and percent equivalents for numbers written in fraction form (proper or improper) or as a mixed number, including repeating decimals. (a) • Represent and determine equivalencies among decimals, percents, fractions (proper or improper), and mixed numbers that have denominators that are 12 or less or factors of 100. (a) • Compare two percents using pictorial representations and symbols (<, ≤, ≥, >, =). (b) • Order no more than four positive rational numbers expressed as fractions (proper or improper), mixed numbers, decimals, and percents (decimals through thousandths, fractions with denominators of 12 or less or factors of 100). Ordering may be in ascending or descending order. (b)

- 6.2 The student will**
a) represent and determine equivalencies among fractions, mixed numbers, decimals, and percents;* and
b) compare and order positive rational numbers.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
<p>are: $\sqrt{25}$, 0.275, $\frac{1}{4}$, 82, 75%, $\frac{22}{5}$, $4.\overline{9}$</p> <ul style="list-style-type: none"> • Students are not expected to know the names of the subsets of the real numbers until grade eight. • Proper fractions, improper fractions, and mixed numbers are terms often used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction may be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., $3\frac{5}{8}$). • Strategies using 0, $\frac{1}{2}$ and 1 as benchmarks can be used to compare fractions. <ul style="list-style-type: none"> – Example: Which is greater, $\frac{4}{7}$ or $\frac{3}{9}$? $\frac{4}{7}$ is greater than $\frac{1}{2}$ because 4, the numerator, represents more than half of 7, the denominator. The denominator tells the number of parts that make the whole. $\frac{3}{9}$ is less than $\frac{1}{2}$ because 3, the numerator, is less than half of 9, the denominator, which tells the number of parts that make the whole. Therefore, $\frac{4}{7} > \frac{3}{9}$. • When comparing two fractions close to 1, use the distance from 1 as your benchmark. <ul style="list-style-type: none"> – Example: Which is greater, $\frac{6}{7}$ or $\frac{8}{9}$? $\frac{6}{7}$ is $\frac{1}{7}$ away from 1 whole. $\frac{8}{9}$ is $\frac{1}{9}$ away from 1 whole. Since, $\frac{1}{9} < \frac{1}{7}$ then $\frac{6}{7}$ is a greater distance away from 1 whole than $\frac{8}{9}$. Therefore, $\frac{6}{7} < \frac{8}{9}$. • Some fractions such as $\frac{1}{8}$ have a decimal representation that is a terminating decimal (e. g., $\frac{1}{8} = 0.125$) and some fractions such as $\frac{2}{9}$ have a decimal representation that does not terminate but continues to repeat (e. g., $\frac{2}{9} = 0.222\dots$). The repeating decimal can be written with ellipses (three dots) as in 0.222... or denoted with a bar above the digits that repeat as in $0.\overline{2}$. 	

6.3

The student will

- a) **identify and represent integers;**
- b) **compare and order integers; and**
- c) **identify and describe absolute value of integers.**

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> • The set of integers includes the set of whole numbers and their opposites $\{\dots-2, -1, 0, 1, 2, \dots\}$. Zero has no opposite and is an integer that is neither positive nor negative. • Integers are used in practical situations, such as temperature (above/below zero), deposits/withdrawals in a checking account, golf (above/below par), time lines, football yardage, positive and negative electrical charges, and altitude (above/below sea level). • Integers should be explored by modeling on a number line and using manipulatives, such as two-color counters, drawings, or algebra tiles. • The opposite of a positive number is negative and the opposite of a negative number is positive. • Positive integers are greater than zero. • Negative integers are less than zero. • A negative integer is always less than a positive integer. • When comparing two negative integers, the negative integer that is closer to zero is greater. • An integer and its opposite are the same distance from zero on a number line. <ul style="list-style-type: none"> – Example: the opposite of 3 is -3 and the opposite of -10 is 10. • On a conventional number line, a smaller number is always located to the left of a larger number (e.g., -7 lies to the left of -3, thus $-7 < -3$; 5 lies to the left of 8 thus 5 is less than 8) • The absolute value of a number is the distance of a number from zero on the number line regardless of direction. Absolute value is represented using the symbol $$ (e.g., $-6 = 6$ and $6 = 6$). • The absolute value of zero is zero. 	<p>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</p> <ul style="list-style-type: none"> • Model integers, including models derived from practical situations. (a) • Identify an integer represented by a point on a number line. (a) • Compare and order integers using a number line. (b) • Compare integers, using mathematical symbols ($<$, \leq, $>$, \geq, $=$). (b) • Identify and describe the absolute value of an integer. (c)

6.4 The student will recognize and represent patterns with whole number exponents and perfect squares.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> • The symbol • can be used in grade six in place of “x” to indicate multiplication. • In exponential notation, the base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. In 8^3, 8 is the base and 3 is the exponent (e.g., $8^3 = 8 \cdot 8 \cdot 8$). • Any real number other than zero raised to the zero power is 1. Zero to the zero power (0^0) is undefined. • A perfect square is a whole number whose square root is an integer (e.g., $36 = 6 \cdot 6 = 6^2$). Zero (a whole number) is a perfect square. • Perfect squares may be represented geometrically as the areas of squares the length of whose sides are whole numbers (e.g., $1 \cdot 1$, $2 \cdot 2$, $3 \cdot 3$, etc.). This can be modeled with grid paper, tiles, geoboards and virtual manipulatives. • The examination of patterns in place value of the powers of 10 in grade six leads to the development of scientific notation in grade seven. 	<p>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</p> <ul style="list-style-type: none"> • Recognize and represent patterns with bases and exponents that are whole numbers. • Recognize and represent patterns of perfect squares not to exceed 20^2, by using grid paper, square tiles, tables, and calculators. • Recognize powers of 10 with whole number exponents by examining patterns in place value.

Focus 6-8 Strand Introduction Computation and Estimation

The computation and estimation strand in grades six through eight focuses on developing conceptual and algorithmic understanding of operations with integers and rational numbers through concrete activities and discussions that bring an understanding as to why procedures work and make sense.

Students develop and refine estimation strategies based on an understanding of number concepts, properties and relationships. The development of problem solving, using operations with integers and rational numbers, builds upon the strategies developed in the elementary grades. Students will reinforce these skills and build on the development of proportional reasoning and more advanced mathematical skills.

Students learn to make sense of the mathematical tools available by making valid judgments about the reasonableness of answers. Students will balance the ability to make precise calculations through the application of the order of operations with knowing when calculations may require estimation to obtain appropriate solutions to practical problems.

6.5

The student will

a) multiply and divide fractions and mixed numbers;*

b) solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions and mixed numbers; and

c) solve multistep practical problems involving addition, subtraction, multiplication, and division of decimals.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> • A fraction can be expressed in simplest form (simplest equivalent fraction) by dividing the numerator and denominator by their greatest common factor. • When the numerator and denominator have no common factors other than 1, then the fraction is in simplest form. • Addition and subtraction are inverse operations as are multiplication and division. • Models for representing multiplication and division of fractions may include arrays, paper folding, repeated addition, repeated subtraction, fraction strips, fraction rods, pattern blocks, and area models. • It is helpful to use estimation to develop computational strategies. <ul style="list-style-type: none"> – Example: $2\frac{7}{8} \cdot \frac{3}{4}$ is about $\frac{3}{4}$ of 3, so the answer is between 2 and 3. • When multiplying a whole number by a fraction such as $3 \cdot \frac{1}{2}$, the meaning is the same as with multiplication of whole numbers: 3 groups the size of $\frac{1}{2}$ of the whole. • When multiplying a fraction by a fraction such as $\frac{2}{3} \cdot \frac{3}{4}$, we are asking for part of a part. • When multiplying a fraction by a whole number such as $\frac{1}{2} \cdot 6$, we are trying to determine a part of the whole. • A multistep problem is a problem that requires two or more steps to solve. • Different strategies can be used to estimate the result of computations and judge the reasonableness of the result. <ul style="list-style-type: none"> – Example: What is an approximate answer for $2.19 \div 0.8$? The answer is around 2 because $2.19 \div 0.8$ is about $2 \div 1 = 2$. • Understanding the placement of the decimal point is important when determining quotients of decimals. Examining patterns with successive decimals provides meaning, such as dividing the dividend by 6, by 0.6, and by 0.06. • Solving multistep problems in the context of practical situations enhances interconnectedness and proficiency with estimation strategies. 	<p>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</p> <ul style="list-style-type: none"> • Demonstrate/model multiplication and division of fractions (proper or improper) and mixed numbers using multiple representations. (a) • Multiply and divide fractions (proper or improper) and mixed numbers. Answers are expressed in simplest form. (a) • Solve single-step and multistep practical problems that involve addition and subtraction with fractions (proper or improper) and mixed numbers, with and without regrouping, that include like and unlike denominators of 12 or less. Answers are expressed in simplest form. (b) • Solve single-step and multistep practical problems that involve multiplication and division with fractions (proper or improper) and mixed numbers that include denominators of 12 or less. Answers are expressed in simplest form. (b) • Solve multistep practical problems involving addition, subtraction, multiplication and division with decimals. Divisors are limited to a three-digit number, with decimal divisors limited to hundredths. (c)

6.5

The student will**a) multiply and divide fractions and mixed numbers;*****b) solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions and mixed numbers; and****c) solve multistep practical problems involving addition, subtraction, multiplication, and division of decimals.****On the state assessment, items measuring this objective are assessed without the use of a calculator.*

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> Examples of practical situations solved by using estimation strategies include shopping for groceries, buying school supplies, budgeting an allowance, and sharing the cost of a pizza or the prize money from a contest. 	

6.6

The student will

- a) **add, subtract, multiply, and divide integers**;^{*}
 b) **solve practical problems involving operations with integers**; and
 c) **simplify numerical expressions involving integers**.^{*}

^{*}On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> • The set of integers is the set of whole numbers and their opposites (e.g., ...-3, -2, -1, 0, 1, 2, 3...). Zero has no opposite and is neither positive nor negative. • Integers are used in practical situations, such as temperature changes (above/below zero), balance in a checking account (deposits/withdrawals), golf, time lines, football yardage, and changes in altitude (above/below sea level). • Concrete experiences in formulating rules for adding, subtracting, multiplying, and dividing integers should be explored by examining patterns using calculators, using a number line, and using manipulatives, such as two-color counters, drawings, or by using algebra tiles. • Sums, differences, products and quotients of integers are either positive, negative, undefined or zero. This may be demonstrated through the use of patterns and models. • The order of operations is a convention that defines the computation order to follow in simplifying an expression. Having an established convention ensures that there is only one correct result when simplifying an expression. • The order of operations is as follows: <ul style="list-style-type: none"> – First, complete all operations within grouping symbols.¹ If there are grouping symbols within other grouping symbols, do the innermost operation first. – Second, evaluate all exponential expressions. – Third, multiply and/or divide in order from left to right. – Fourth, add and/or subtract in order from left to right. <p>¹Parentheses (), absolute value (e.g., 3(-5 + 2)), and the division bar (e.g., $\frac{3+4}{5+6}$) should be treated as grouping symbols.</p> • Expressions are simplified using the order of operations and applying the properties of real numbers. Students should use the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of a, b, or c in this standard): <ul style="list-style-type: none"> – Commutative property of addition: $a + b = b + a$. – Commutative property of multiplication: $a \cdot b = b \cdot a$. – Associative property of addition: $(a + b) + c = a + (b + c)$. 	<p>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</p> <ul style="list-style-type: none"> • Model addition, subtraction, multiplication and division of integers using pictorial representations or concrete manipulatives. (a) • Add, subtract, multiply, and divide two integers. (a) • Solve practical problems involving addition, subtraction, multiplication, and division with integers. (b) • Use the order of operations and apply the properties of real numbers to simplify numerical expressions involving more than two integers. Expressions should not include braces { } or brackets [], but may contain absolute value bars . Simplification will be limited to three operations, which may include simplifying a whole number raised to an exponent of 1, 2 or 3. (c)

6.6

The student will

- a) **add, subtract, multiply, and divide integers;***
 b) **solve practical problems involving operations with integers; and**
 c) **simplify numerical expressions involving integers.***

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> - Associative property of multiplication: $(ab)c = a(bc)$. - Subtraction and division are neither commutative nor associative. - Distributive property (over addition/subtraction): $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$. - Identity property of addition (additive identity property): $a + 0 = a$ and $0 + a = a$. - Identity property of multiplication (multiplicative identity property): $a \cdot 1 = a$ and $1 \cdot a = a$. - The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by one is the number. There are no identity elements for subtraction and division. - Inverse property of addition (additive inverse property): $a + (-a) = 0$ and $(-a) + a = 0$. - Multiplicative property of zero: $a \cdot 0 = 0$ and $0 \cdot a = 0$ - Substitution property: If $a = b$ then b can be substituted for a in any expression, equation or inequality. • The power of a number represents repeated multiplication of the number (e.g., $8^3 = 8 \cdot 8 \cdot 8$). The base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. In the example, 8 is the base, and 3 is the exponent. • Any number, except zero, raised to the zero power is 1. Zero to the zero power (0^0) is undefined. 	

Focus 6-8 Strand Introduction Measurement and Geometry

Measurement and geometry in the middle grades provide a natural context and connection among many mathematical concepts. Students expand informal experiences with geometry and measurement in the elementary grades and develop a solid foundation for further exploration of these concepts in high school. Spatial reasoning skills are essential to the formal inductive and deductive reasoning skills required in subsequent mathematics learning.

Students develop measurement skills through exploration and estimation. Physical exploration to determine length, weight/mass, liquid volume/capacity, and angle measure are essential to develop a conceptual understanding of measurement. Students examine perimeter, area, and volume, using concrete materials and practical situations. Students focus their study of surface area and volume on rectangular prisms, cylinders, square-based pyramids, and cones.

Students learn geometric relationships by visualizing, comparing, constructing, sketching, measuring, transforming, and classifying geometric figures. A variety of tools such as geoboards, pattern blocks, dot paper, patty paper, and geometry software provide experiences that help students discover geometric concepts. Students describe, classify, and compare plane and solid figures according to their attributes. They develop and extend understanding of geometric transformations in the coordinate plane.

Students apply their understanding of perimeter and area from the elementary grades in order to build conceptual understanding of the surface area and volume of prisms, cylinders, square-based pyramids, and cones. They use visualization, measurement, and proportional reasoning skills to develop an understanding of the effect of scale change on distance, area, and volume. They develop and reinforce proportional reasoning skills through the study of similar figures.

Students explore and develop an understanding of the Pythagorean Theorem. Understanding how the Pythagorean Theorem can be applied in practical situations has a far-reaching impact on subsequent mathematics learning and life experiences.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

Level 0: Pre-recognition. Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

Level 1: Visualization. Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during kindergarten and grade one.)

Level 2: Analysis. Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades two and three.)

Level 3: Abstraction. Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades five and six and fully attain it before taking algebra.)

Level 4: Deduction. Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students should be able to supply reasons for steps in a proof. (Students should transition to this level before taking geometry.)

- 6.7 The student will**
- derive π (π);
 - solve problems, including practical problems, involving circumference and area of a circle; and
 - solve problems, including practical problems, involving area and perimeter of triangles and rectangles.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> The value of pi (π) is the ratio of the circumference of a circle to its diameter. Thus, the circumference of a circle is proportional to its diameter. The calculation of determining area and circumference may vary depending upon the approximation for pi. Common approximations for π include 3.14, $\frac{22}{7}$, or the pi (π) button on a calculator. Experiences in deriving the formulas for area, perimeter, and volume using manipulatives such as tiles, one-inch cubes, graph paper, geoboards, or tracing paper, promote an understanding of the formulas and their use. Perimeter is the path or distance around any plane figure. The perimeter of a circle is called the circumference. The circumference of a circle is about three times the measure of its diameter. The circumference of a circle is computed using $C = \pi d$ or $C = 2\pi r$, where d is the diameter and r is the radius of the circle. The area of a closed curve is the number of nonoverlapping square units required to fill the region enclosed by the curve. The area of a circle is computed using the formula $A = \pi r^2$, where r is the radius of the circle. The perimeter of a square whose side measures s can be determined by multiplying 4 by s ($P = 4s$), and its area can be determined by squaring the length of one side ($A = s^2$). The perimeter of a rectangle can be determined by computing the sum of twice the length and twice the width ($P = 2l + 2w$, or $P = 2(l + w)$), and its area can be determined by computing the product of the length and the width ($A = lw$). The perimeter of a triangle can be determined by computing the sum of the side lengths ($P = a + b + c$), and its area can be determined by computing $\frac{1}{2}$ the product of base and the height ($A = \frac{1}{2}bh$). 	<p>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</p> <ul style="list-style-type: none"> Derive an approximation for pi (3.14 or $\frac{22}{7}$) by gathering data and comparing the circumference to the diameter of various circles, using concrete materials or computer models. (a) Solve problems, including practical problems, involving circumference and area of a circle when given the length of the diameter or radius. (b) Solve problems, including practical problems, involving area and perimeter of triangles and rectangles. (c)

6.8

The student will

- a) **identify the components of the coordinate plane; and**
- b) **identify the coordinates of a point and graph ordered pairs in a coordinate plane.**

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> • In a coordinate plane, the coordinates of a point are typically represented by the ordered pair (x, y), where x is the first coordinate and y is the second coordinate. • Any given point is defined by only one ordered pair in the coordinate plane. • The grid lines on a coordinate plane are perpendicular. • The axes of the coordinate plane are the two intersecting perpendicular lines that divide it into its four quadrants. The x-axis is the horizontal axis and the y-axis is the vertical axis. • The quadrants of a coordinate plane are the four regions created by the two intersecting perpendicular lines (x- and y-axes). Quadrants are named in counterclockwise order. The signs on the ordered pairs for quadrant I are $(+, +)$; for quadrant II, $(-, +)$; for quadrant III, $(-, -)$; and for quadrant IV, $(+, -)$. • In a coordinate plane, the origin is the point at the intersection of the x-axis and y-axis; the coordinates of this point are $(0, 0)$. • For all points on the x-axis, the y-coordinate is 0. For all points on the y-axis, the x-coordinate is 0. • The coordinates may be used to name the point. (e.g., the point $(2, 7)$). It is not necessary to say “the point whose coordinates are $(2, 7)$.” The first coordinate tells the location or distance of the point to the left or right of the y-axis and the second coordinate tells the location or distance of the point above or below the x-axis. For example, $(2, 7)$ is two units to the right of the y-axis and seven units above the x-axis. • Coordinates of points having the same x-coordinate are located on the same vertical line. Foreexample, $(2, 4)$ and $(2, -3)$ are both two units to the right of the y-axis and are vertically seven units from each other. • Coordinates of points having the same y-coordinate are located on the same horizontal line. For example, $(-4, -2)$ and $(2, -2)$ are both two units below the x-axis and are horizontally six units from each other. 	<p>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</p> <ul style="list-style-type: none"> • Identify and label the axes, origin, and quadrants of a coordinate plane. (a) • Identify the quadrant or the axis on which a point is positioned by examining the coordinates (ordered pair) of the point. Ordered pairs will be limited to coordinates expressed as integers. (a) • Graph ordered pairs in the four quadrants and on the axes of a coordinate plane. Ordered pairs will be limited to coordinates expressed as integers. (b) • Identify ordered pairs represented by points in the four quadrants and on the axes of the coordinate plane. Ordered pairs will be limited to coordinates expressed as integers. (b) • Relate the coordinates of a point to the distance from each axis and relate the coordinates of a single point to another point on the same horizontal or vertical line. Ordered pairs will be limited to coordinates expressed as integers. (b) • Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to determine the length of a side joining points with the same first coordinate or the same second coordinate. Ordered pairs will be limited to coordinates expressed as integers. Apply these techniques in the context of solving practical and mathematical problems. (b)

6.9 The student will determine congruence of segments, angles, and polygons.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> The symbol for congruency is \cong. Congruent figures have exactly the same size and the same shape. Line segments are congruent if they have the same length. Angles are congruent if they have the same measure. Congruent polygons have an equal number of sides, and all the corresponding sides and angles are congruent. <ul style="list-style-type: none"> Examples: <div data-bbox="331 526 1024 922" data-label="Diagram"> </div> <ul style="list-style-type: none"> A polygon is a closed plane figure composed of at least three line segments that do not cross. A regular polygon has congruent sides and congruent interior angles. The number of lines of symmetry of a regular polygon is equal to the number of sides of the polygon. A line of symmetry divides a figure into two congruent parts, each of which is the mirror image of the other. Lines of symmetry are not limited to horizontal and vertical lines. Noncongruent figures may have the same shape but not the same size. Students should be familiar with geometric markings in figures to indicate congruence of sides and angles and to indicate parallel sides. An equal number of hatch (hash) marks indicate that those sides are equal in length. An equal number of arrows indicate that those sides are parallel. An equal number of angle curves indicate that those angles have the same measure. See the diagram below. 	<p>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</p> <ul style="list-style-type: none"> Identify regular polygons. Draw lines of symmetry to divide regular polygons into two congruent parts. Determine the congruence of segments, angles, and polygons given their properties. Determine whether polygons are congruent or noncongruent according to the measures of their sides and angles.

6.9 The student will determine congruence of segments, angles, and polygons.

Understanding the Standard	Essential Knowledge and Skills
<div data-bbox="191 298 1104 537" data-label="Diagram"> </div> <ul data-bbox="92 553 1224 688" style="list-style-type: none"> • The determination of the congruence or noncongruence of two figures can be accomplished by placing one figure on top of the other or by comparing the measurements of all corresponding sides and angles. • Construction of congruent line segments, angles, and polygons helps students understand congruency. 	

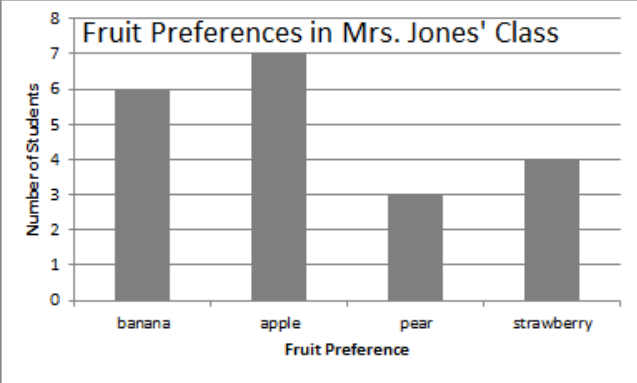
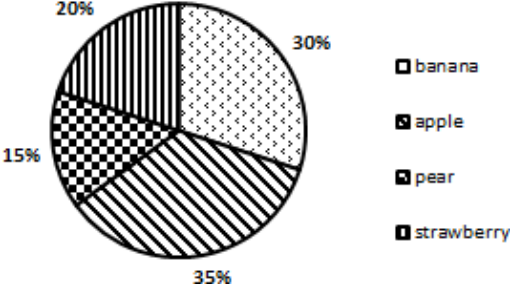
Focus 6–8 Strand Introduction Probability and Statistics

In the middle grades, students develop an awareness of the power of data analysis and the application of probability through fostering their natural curiosity about data and making predictions.

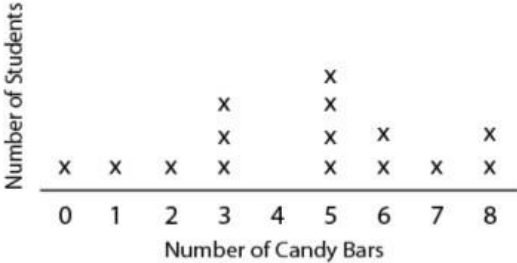
The exploration of various methods of data collection and representation allows students to become effective at using different types of graphs to represent different types of data. Students use measures of center and dispersion to analyze and interpret data.

Students integrate their understanding of rational numbers and proportional reasoning into the study of statistics and probability. Through experiments and simulations, students build on their understanding of the Fundamental Counting Principle from elementary mathematics to learn more about probability in the middle grades.




- 6.10** The student, given a practical situation, will
- represent data in a circle graph;
 - make observations and inferences about data represented in a circle graph; and
 - compare circle graphs with the same data represented in bar graphs, pictographs, and line plots.

Understanding the Standard	Essential Knowledge and Skills										
<p>• Circle graphs are used for data showing a relationship of the parts to the whole.</p> <p>– Example: the favorite fruit of 20 students in Mrs. Jones class was recorded in the</p> <table border="1" data-bbox="151 529 504 721"> <thead> <tr> <th>Fruit Preference</th> <th># of students</th> </tr> </thead> <tbody> <tr> <td>banana</td> <td>6</td> </tr> <tr> <td>apple</td> <td>7</td> </tr> <tr> <td>pear</td> <td>3</td> </tr> <tr> <td>strawberry</td> <td>4</td> </tr> </tbody> </table> <p>table. Compare the same data displayed in both a circle graph and a bar graph.</p>  <p>• Circle graphs can represent percent or frequency.</p>	Fruit Preference	# of students	banana	6	apple	7	pear	3	strawberry	4	<p>Fruit Preferences in Mrs. Jones' Class</p>  <p>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</p> <ul style="list-style-type: none"> Collect, organize and represent data in a circle graph. The number of data values should be limited to allow for comparisons that have denominators of 12 or less or those that are factors of 100 (e.g., in a class of 20 students, 7 choose apples as a favorite fruit, so the comparison is 7 out of 20, $\frac{7}{20}$ or 35%). (a) Make observations and inferences about data represented in a circle graph. (b) Compare data represented in a circle graph with the same data represented in bar graphs, pictographs, and line plots. (c)
Fruit Preference	# of students										
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- 6.10 The student, given a practical situation, will**
- represent data in a circle graph;**
 - make observations and inferences about data represented in a circle graph; and**
 - compare circle graphs with the same data represented in bar graphs, pictographs, and line plots.**

Understanding the Standard	Essential Knowledge and Skills																				
<ul style="list-style-type: none"> • Circle graphs are not useful for representing data with large numbers of categories. • Teachers should be reasonable about the selection of data values. The number of data values can affect how a circle graph is constructed (e.g., 10 out of 25 would be 40%, but 7 out of 9 would be $77.\overline{7}\%$, making the construction of a circle graph more complex). Students should have experience constructing circle graphs, but a focus should be placed on the analysis of circle graphs. • Students are not expected to construct circle graphs by multiplying the percentage of data in a category by 360° in order to determine the central angle measure. Limiting comparisons to fraction parameters noted will assist students in constructing circle graphs. • To collect data for any problem situation, an experiment can be designed, a survey can be conducted, or other data-gathering strategies can be used. The data can be organized, displayed, analyzed, and interpreted to solve the problem. • Categorical data can be sorted into groups or categories while numerical data are values or observations that can be measured. For example, types of fish caught would be categorical data while weights of fish caught would be numerical data. • Different types of graphs can be used to display categorical data. The way data are displayed often depends on what someone is trying to communicate. <ul style="list-style-type: none"> – A line plot is used for categorical and discrete numerical data and is used to show frequency of data on a number line. It is a simple way to organize data. Example: <div style="text-align: center; margin: 10px 0;"> <p>Candy Bars</p>  <table border="1" style="margin: 0 auto; border-collapse: collapse;"> <thead> <tr> <th>Number of Candy Bars</th> <th>Number of Students</th> </tr> </thead> <tbody> <tr><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td></tr> <tr><td>2</td><td>1</td></tr> <tr><td>3</td><td>2</td></tr> <tr><td>4</td><td>1</td></tr> <tr><td>5</td><td>3</td></tr> <tr><td>6</td><td>2</td></tr> <tr><td>7</td><td>1</td></tr> <tr><td>8</td><td>1</td></tr> </tbody> </table> </div> <ul style="list-style-type: none"> – A bar graph is used for categorical and discrete numerical data (e.g., number of months or number of people with a particular eye color) and is used to show comparisons. 	Number of Candy Bars	Number of Students	0	1	1	1	2	1	3	2	4	1	5	3	6	2	7	1	8	1	
Number of Candy Bars	Number of Students																				
0	1																				
1	1																				
2	1																				
3	2																				
4	1																				
5	3																				
6	2																				
7	1																				
8	1																				

- 6.10** The student, given a practical situation, will
- a) represent data in a circle graph;
 - b) make observations and inferences about data represented in a circle graph; and
 - c) compare circle graphs with the same data represented in bar graphs, pictographs, and line plots.

Understanding the Standard	Essential Knowledge and Skills								
<p>– A pictograph is mainly used to show categorical data. Pictographs are used to show frequency and compare items. However, the use of partial pictures can give misleading information.</p> <ul style="list-style-type: none"> ○ Example: <p style="text-align: center;">The Types of Pets We Have</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th style="text-align: center;">Cat</th> <th style="text-align: center;">Dog</th> <th style="text-align: center;">Horse</th> <th style="text-align: center;">Fish</th> </tr> </thead> <tbody> <tr> <td style="text-align: center; vertical-align: middle;">  </td> <td></td> <td></td> <td></td> </tr> </tbody> </table> <p style="text-align: center;">= 1 student</p> <ul style="list-style-type: none"> • A circle graph is used for categorical and discrete numerical data. Circle graphs are used to show a relationship of the parts to a whole. • All graphs must include a title, percent or number labels for data categories, and a key. A key is essential to explain how to read the graph. A title is essential to explain what the graph represents. • A scale should be chosen that is appropriate for the data values being represented. • Comparisons, predictions, and inferences are made by examining characteristics of a data set displayed in a variety of graphical representations to draw conclusions. • The information displayed in different graphs may be examined to determine how data are or are not related, differences between characteristics (comparisons), trends that suggest what new data might be like (predictions), and/or “what could happen if” (inferences). 	Cat	Dog	Horse	Fish					
Cat	Dog	Horse	Fish						
									

- 6.11 The student will**
a) represent the mean of a data set graphically as the balance point; and
b) determine the effect on measures of center when a single value of a data set is added, removed, or changed.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> • Categorical data can be sorted into groups or categories while numerical data are values or observations that can be measured. For example, types of fish caught would be categorical data while weights of fish caught would be numerical data. • Measures of center are types of averages for a data set. They represent numbers that describe a data set. Mean, median, and mode are measures of center that are useful for describing the average for different situations. <ul style="list-style-type: none"> – Mean may be appropriate for sets of data where there are no values much higher or lower than those in the rest of the data set. – Median is a good choice when data sets have a couple of values much higher or lower than most of the others. – Mode is a good descriptor to use when the set of data has some identical values, when data is non-numeric (categorical) or when data reflects the most popular item. • Mean can be defined as the point on a number line where the data distribution is balanced. This requires that the sum of the distances from the mean of all the points above the mean is equal to the sum of the distances from the mean of all the data points below the mean. This is the concept of mean as the balance point. <ul style="list-style-type: none"> – Example: Given the data set: <div style="text-align: center;">2, 3, 4, 7</div> The mean value of 4 can be represented on a number line as the balance point: • The mean can also be found by calculating the numerical average of the data set. • In grade five mathematics, mean is defined as fair share. • Defining mean as the balance point is a prerequisite for understanding standard deviation, which is addressed in high school level mathematics. • The median is the middle value of a data set in ranked order. If there are an odd number of pieces of data, the median is the middle value in ranked order. If there is an even number of pieces of data, the median is the numerical average of the two middle values. • The mode is the piece of data that occurs most frequently. If no value occurs more often than any other, there is no mode. If there is more than one value that occurs most often, all these most-frequently-occurring values are modes. When there are exactly two modes, the data set is bimodal. 	<p>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</p> <ul style="list-style-type: none"> • Represent the mean of a set of data graphically as the balance point represented in a line plot. (a) • Determine the effect on measures of center when a single value of a data set is added, removed, or changed. (b)

Focus 6–8 Strand Introduction Patterns, Functions, and Algebra

Patterns, functions and algebra become a larger mathematical focus in the middle grades as students extend their knowledge of patterns developed in the elementary grades.

Students make connections between the numeric concepts of ratio and proportion and the algebraic relationships that exist within a set of equivalent ratios. Students use variable expressions to represent proportional relationships between two quantities and begin to connect the concept of a constant of proportionality to rate of change and slope. Representation of relationships between two quantities using tables, graphs, equations, or verbal descriptions allow students to connect their knowledge of patterns to the concept of functional relationships. Graphing linear equations in two variables in the coordinate plane is a focus of the study of functions which continues in high school mathematics.

Students learn to use algebraic concepts and terms appropriately. These concepts and terms include *variable*, *term*, *coefficient*, *exponent*, *expression*, *equation*, *inequality*, *domain*, and *range*. Developing a beginning knowledge of algebra is a major focus of mathematics learning in the middle grades. Students learn to solve equations by using concrete materials. They expand their skills from one-step to multistep equations and inequalities through their application in practical situations.

6.12

The student will

- a) represent a proportional relationship between two quantities, including those arising from practical situations;
- b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;
- c) determine whether a proportional relationship exists between two quantities; and
- d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> • A ratio is a comparison of any two quantities. A ratio is used to represent relationships within a quantity and between quantities. • Equivalent ratios arise by multiplying each value in a ratio by the same constant value. For example, the ratio of 4:2 would be equivalent to the ratio 8:4, since each value in the first ratio could be multiplied by 2 to obtain the second ratio. • A proportional relationship consists of two quantities where there exists a constant number (constant of proportionality) such that each measure in the first quantity multiplied by this constant gives the corresponding measure in the second quantity. • Proportional thinking requires students to thinking multiplicatively, versus additively. The relationship between two quantities could be additive (i.e., one quantity is a result of adding a value to the other quantity) or multiplicative (i.e., one quantity is the result of multiplying the other quantity by a value). Therefore, it is important to use practical situations to model proportional relationships, because context can help students to see the relationship. Students will explore algebraic representations of additive relationships in grade seven. <ul style="list-style-type: none"> – Example: <ul style="list-style-type: none"> ○ In the additive relationship, y is the result of adding 8 to x. ○ In the multiplicative relationship, y is the result of multiplying 5 times x. ○ The ordered pair (2, 10) is a quantity in both relationships; however, the relationship evident between the other quantities in the table, discerns between additive or multiplicative. • Students have had experiences with tables of values (input/output tables that are additive and multiplicative) in elementary grades. • A ratio table is a table of values representing a proportional relationship that includes pairs of values that represent equivalent rates or ratios. A constant exists that can be multiplied by the measure of one quantity to get the measure of the other quantity for every ratio pair. The same proportional relationship exists between each pair of quantities in a ratio table. <ul style="list-style-type: none"> – Example: Given that the ratio of y to x in a proportional relationship is 8:4, create a ratio table that includes three additional equivalent ratios. 	<p>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</p> <ul style="list-style-type: none"> • Make a table of equivalent ratios to represent a proportional relationship between two quantities, when given a ratio. (a) • Make a table of equivalent ratios to represent a proportional relationship between two quantities, when given a practical situation. (a) • Identify the unit rate of a proportional relationship represented by a table of values or a verbal description, including those represented in a practical situation. Unit rates are limited to positive values. (b) • Determine a missing value in a ratio table that represents a proportional relationship between two quantities using a unit rate. Unit rates are limited to positive values. (b) • Determine whether a proportional relationship exists between two quantities, when given a table of values or a verbal description, including those represented in a practical situation. Unit rates are limited to positive values. (c) • Determine whether a proportional relationship exists between two quantities given a graph of ordered pairs. Unit rates are limited to positive values. (c) • Make connections between and among multiple representations of the same proportional relationship using verbal descriptions, ratio tables, and graphs. Unit rates are limited to positive values. (d)

6.12

The student will

- represent a proportional relationship between two quantities, including those arising from practical situations;
- determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;
- determine whether a proportional relationship exists between two quantities; and
- make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

Understanding the Standard	Essential Knowledge and Skills										
<p>Students have had experience with tables of values (input/output tables) in elementary grades and the concept of a ratio table should be connected to their prior knowledge of representing number patterns in tables.</p> <ul style="list-style-type: none"> A rate is a ratio that involves two different units and how they relate to each other. Relationships between two units of measure are also rates (e.g., inches per foot). A unit rate describes how many units of the first quantity of a ratio correspond to one unit of the second quantity. <ul style="list-style-type: none"> Example: If it costs \$10 for 5 items at a store (a ratio of 10:5 comparing cost to the number of items), then the unit rate would be \$2.00/per item (a ratio of 2:1 comparing cost to number of items). <table border="1" data-bbox="348 873 970 1016"> <tbody> <tr> <td># of items (x)</td> <td>1</td> <td>2</td> <td>5</td> <td>10</td> </tr> <tr> <td>Cost in \$ (y)</td> <td>\$2.00</td> <td>\$4.00</td> <td>\$10.00</td> <td>\$20.00</td> </tr> </tbody> </table> <ul style="list-style-type: none"> Any ratio can be converted into a unit rate by writing the ratio as a fraction and then dividing the numerator and denominator each by the value of the denominator. Example: It costs \$8 for 16 gourmet cookies at a bake sale. What is the price per cookie (unit rate) represented by this situation? $\frac{8}{16} = \frac{8 \div 16}{16 \div 16} = \frac{0.5}{1}$ So, it would cost \$0.50 per cookie, which would be the unit rate. <ul style="list-style-type: none"> Example: $\frac{8}{16}$ and 40 to 10 are ratios, but are not unit rates. However, $\frac{0.5}{1}$ and 4 to 1 are unit rates. Students in grade six should build a conceptual understanding of proportional relationships and unit rates before moving to more abstract representations and complex computations in higher grade levels. Students are not expected to use formal calculations for slope and unit rates (e.g., slope formula) in grade six. 	# of items (x)	1	2	5	10	Cost in \$ (y)	\$2.00	\$4.00	\$10.00	\$20.00	
# of items (x)	1	2	5	10							
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6.12

The student will

- represent a proportional relationship between two quantities, including those arising from practical situations;
- determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;
- determine whether a proportional relationship exists between two quantities; and
- make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

Understanding the Standard	Essential Knowledge and Skills
<p>– Example of a proportional relationship:</p> <p>Ms. Cochran is planning a year-end pizza party for her students. Ace Pizza offers free delivery and charges \$8 for each medium pizza. This ratio table represents the cost (y) per number of pizzas ordered (x).</p> <p>In this relationship, the ratio of y (cost in \$) to x (number of pizzas) in each ordered pair is the same:</p> $\frac{8}{1} = \frac{16}{2} = \frac{24}{3} = \frac{32}{4}$ <p>– Example of a non-proportional relationship:</p> <p>Uptown Pizza sells medium pizzas for \$7 each but charges a \$3 delivery fee per order. This table represents the cost per number of pizzas ordered.</p> <p>The ratios represented in the table above are not equivalent.</p> <p>In this relationship, the ratio of y to x in each ordered pair is not the same:</p> $\frac{10}{1} \neq \frac{17}{2} \neq \frac{24}{3} \neq \frac{31}{4}$ <p>Other non-proportional relationships will be studied in later mathematics courses.</p> <ul style="list-style-type: none"> Proportional relationships can be described verbally using the phrases “for each,” “for every,” and “per.” Proportional relationships involve collections of pairs of equivalent ratios that may be graphed in the coordinate plane. The graph of a proportional relationship includes ordered pairs (x, y) that represent pairs of values that may be represented in a ratio table. Proportional relationships can be expressed using verbal descriptions, tables, and graphs. <ul style="list-style-type: none"> Example: (verbal description) To make a drink, mix 1 liter of syrup with 3 liters of water. If x represents how many liters of syrup are in the mixture and y represents how many liters of water are in the mixture, this proportional relationship can be represented using a ratio table: 	

6.12

The student will

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- b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;
- c) determine whether a proportional relationship exists between two quantities; and
- d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

Understanding the Standard	Essential Knowledge and Skills
<p>The ratio of the amount of water (y) to the amount of syrup (x) is 3:1. Additionally, the proportional relationship may be graphed using the ordered pairs in the table.</p> <ul style="list-style-type: none"> • The representation of the ratio between two quantities may depend upon the order in which the quantities are being compared. <ul style="list-style-type: none"> – Example: In the mixture example above, we could also compare the ratio of the liters of syrup per liters of water, as shown: 	

6.12

The student will

- a) represent a proportional relationship between two quantities, including those arising from practical situations;
- b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;
- c) determine whether a proportional relationship exists between two quantities; and
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Understanding the Standard	Essential Knowledge and Skills
<p>In this comparison, the ratio of the amount of syrup (y) to the amount of water (x) would be 1:3.</p> <p>The graph of this relationship could be represented by:</p> <p>Students should be aware of how the order in which quantities are compared affects the way in which the relationship is represented as a table of equivalent ratios or as a graph.</p> <ul style="list-style-type: none"> • Double number line diagrams can also be used to represent proportional relationships and create collections of pairs of equivalent ratios. <ul style="list-style-type: none"> – Example: <p>In this proportional relationship, there are three liters of water for each liter of syrup represented on the number lines.</p> • A graph representing a proportional relationship includes ordered pairs that lie in a straight line that, if extended, would pass through $(0, 0)$, creating a pattern of horizontal and vertical increases. The context of the problem and the type of data being represented by the graph must be considered when determining whether the points are to be connected by a straight line on the graph. <ul style="list-style-type: none"> – Example of the graph of a non-proportional relationship: <p>The relationship of distance (y) to time (x) is non-proportional. The ratio of y to x for each ordered pair is not equivalent. That is,</p> $\frac{11}{8} \neq \frac{9}{6} \neq \frac{5}{4} \neq \frac{3}{2} \neq \frac{1}{0}$ <p>The points of the graph do not lie in a straight line. Additionally, the line does not pass through</p> 	

6.12

The student will

- a) represent a proportional relationship between two quantities, including those arising from practical situations;
- b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;
- c) determine whether a proportional relationship exists between two quantities; and
- d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

Understanding the Standard	Essential Knowledge and Skills												
<p>the point $(0, 0)$, thus the relationship of y to x cannot be considered proportional.</p> <ul style="list-style-type: none"> • Practical situations that model proportional relationships can typically be represented by graphs in the first quadrant, since in most cases the values for x and y are not negative. • Unit rates are not typically negative in practical situations involving proportional relationships. • A unit rate could be used to find missing values in a ratio table. <ul style="list-style-type: none"> – Example: A store advertises a price of \$25 for 5 DVDs. What would be the cost to purchase 2 DVDs? 3 DVDs? 4 DVDs? <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th># DVDs</th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <th>Cost</th> <td>\$5</td> <td>?</td> <td>?</td> <td>?</td> <td>\$25</td> </tr> </tbody> </table> <p>The ratio of \$25 per 5 DVDs is also equivalent to a ratio of \$5 per 1 DVD, which would be the unit rate for this relationship. This unit rate could be used to find the missing value of the ratio table above. If we multiply the number of DVDs by a constant multiplier of 5 (the unit rate) we obtain the total cost. Thus, 2 DVDs would cost \$10, 3 DVDs would cost \$15, and 4 DVDs would cost \$20.</p>	# DVDs	1	2	3	4	5	Cost	\$5	?	?	?	\$25	
# DVDs	1	2	3	4	5								
Cost	\$5	?	?	?	\$25								

6.13 The student will solve one-step linear equations in one variable, including practical problems that require the solution of a one-step linear equation in one variable.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> • A one-step linear equation may include, but not be limited to, equations such as the following: $2x = 5$; $y - 3 = -6$; $\frac{1}{5}x = -3$; $a - (-4) = 11$. • A variety of concrete materials such as colored chips, algebra tiles, or weights on a balance scale may be used to model solving equations in one variable. • An expression is a representation of quantity. It may contain numbers, variables, and/or operation symbols. It does not have an “equal sign (=)” (e.g., $\frac{3}{4}$, $5x$, $140 - 38.2$, $18 \cdot 21$, $5 + x$.) • An expression that contains a variable is a variable expression. A variable expression is like a phrase: As a phrase does not have a verb, so an expression does not have an “equal sign (=)”. An expression cannot be solved. • A verbal expression can be represented by a variable expression. Numbers are used when they are known; variables are used when the numbers are unknown. Example, the verbal expression “a number multiplied by 5” could be represented by the variable expression “$n \cdot 5$” or “$5n$.” • An algebraic expression is a variable expression that contains at least one variable (e.g., $x - 3$). • A verbal sentence is a complete word statement (e.g., “The sum of a number and two is five” could be represented by “$n + 2 = 5$”). • An algebraic equation is a mathematical statement that says that two expressions are equal (e.g., $2x = 7$). • A term is a number, variable, product, or quotient in an expression of sums and/or differences. In $7x^2 + 5x - 3$, there are three terms, $7x^2$, $5x$, and 3. • A coefficient is the numerical factor in a term. Example: in the term $3xy^2$, 3 is the coefficient; in the term z, 1 is the coefficient. • An equation is a mathematical sentence stating that two expressions are equal. • A variable is a symbol used to represent an unknown quantity. • The solution to an equation is a value that makes it a true statement. Many equations have one solution and are represented as a point on a number line. Solving an equation or inequality involves a process of determining which value(s) from a specified set, if any, make the equation or inequality a true statement. Substitution can be used to determine whether a given value(s) makes 	<p>The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to</p> <ul style="list-style-type: none"> • Identify examples of the following algebraic vocabulary: equation, variable, expression, term, and coefficient. • Represent and solve one-step linear equations in one variable, using a variety of concrete materials such as colored chips, algebra tiles, or weights on a balance scale. • Apply properties of real numbers and properties of equality to solve a one-step equation in one variable. Coefficients are limited to integers and unit fractions. Numeric terms are limited to integers. • Confirm solutions to one-step linear equations in one variable. • Write verbal expressions and sentences as algebraic expressions and equations. • Write algebraic expressions and equations as verbal expressions and sentences. • Represent and solve a practical problem with a one-step linear equation in one variable.

6.13 The student will solve one-step linear equations in one variable, including practical problems that require the solution of a one-step linear equation in one variable.

Understanding the Standard	Essential Knowledge and Skills
<p>an equation or inequality true.</p> <ul style="list-style-type: none"> • Properties of real numbers and properties of equality can be used to solve equations, justify equation solutions, and express simplification. Students should use the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of a, b, or c in this standard). <ul style="list-style-type: none"> – Commutative property of addition: $a + b = b + a$. – Commutative property of multiplication: $a \cdot b = b \cdot a$. – Subtraction and division are neither commutative nor associative. – Identity property of addition (additive identity property): $a + 0 = a$ and $0 + a = a$. – Identity property of multiplication (multiplicative identity property): $a \cdot 1 = a$ and $1 \cdot a = a$. – The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by one is the number. There are no identity elements for subtraction and division. – Inverses are numbers that combine with other numbers and result in identity elements (e.g., $5 + (-5) = 0$; $\frac{1}{5} \cdot 5 = 1$). – Inverse property of addition (additive inverse property): $a + (-a) = 0$ and $(-a) + a = 0$. – Inverse property of multiplication (multiplicative inverse property): $a \cdot \frac{1}{a} = 1$ and $\frac{1}{a} \cdot a = 1$. – Zero has no multiplicative inverse. – Multiplicative property of zero: $a \cdot 0 = 0$ and $0 \cdot a = 0$. – Division by zero is not a possible mathematical operation. It is undefined. – Addition property of equality: If $a = b$, then $a + c = b + c$. – Subtraction property of equality: If $a = b$, then $a - c = b - c$. – Multiplication property of equality: If $a = b$, then $a \cdot c = b \cdot c$. – Division property of equality: If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$. – Substitution property: If $a = b$ then b can be substituted for a in any expression, equation or inequality. 	

6.14

The student will

a) represent a practical situation with a linear inequality in one variable; and

b) solve one-step linear inequalities in one variable, involving addition or subtraction, and graph the solution on a number line.

Understanding the Standard	Essential Knowledge and Skills
<ul style="list-style-type: none"> • The solution set to an inequality is the set of all numbers that make the inequality true. • Inequalities can represent practical situations. Example: Jaxon works at least 4 hours per week mowing lawns. Write an inequality representing this situation and graph the solution. $x \geq 4 \text{ or } 4 \leq x$ Students might then be asked: “Would Jaxon ever work 3 hours in a week? 6 hours?” • The variable in an inequality may represent values that are limited by the context of the problem or situation. Example: if the variable represents all children in a classroom who are taller than 36 inches, the variable will be limited to have a minimum and maximum value based on the heights of the children. Students are not expected to represent these situations with a compound inequality (e.g., $36 < x < 70$) but only recognize that the values satisfying the single inequality ($x > 36$) will be limited by the context of the situation. • Inequalities using the $<$ or $>$ symbols are represented on a number line with an open circle on the number and a shaded line over the solution set. Example: When graphing $x < 4$, use an open circle above the 4 to indicate that the 4 is not included. • Inequalities using the \leq or \geq symbols are represented on a number line with a closed circle on the number and shaded line in the direction of the solution set. Example: When graphing $x \geq 4$ fill in the circle above the 4 to indicate that the 4 is included. • It is important for students to see inequalities written with the variable before the inequality symbol and after. Example: $x > 5$ is not the same relationship as $5 > x$. However, $x > 5$ is the same relationship as $5 < x$. • A one-step linear inequality may include, but not be limited to, inequalities such as the following: $2 + x > 5$; $y - 3 \leq -6$; $a - (-4) \geq 11$. • Solving an equation or inequality involves a process of determining which value(s) from a specified set, if any, make the equation or inequality a true statement. Substitution can be used to determine whether a given value(s) makes an equation or inequality true. • Properties of real numbers and properties of inequality can be used to solve inequalities, justify solutions, 	<p>The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to</p> <ul style="list-style-type: none"> • Given a verbal description, represent a practical situation with a one-variable linear inequality. (a) • Apply properties of real numbers and the addition or subtraction property of inequality to solve a one-step linear inequality in one variable, and graph the solution on a number line. Numeric terms being added or subtracted from the variable are limited to integers. (b) • Given the graph of a linear inequality with integers, represent the inequality two different ways (e.g., $x < -5$ or $-5 > x$) using symbols. (b) • Identify a numerical value(s) that is part of the solution set of a given inequality. (a, b)

6.14

The student will

- a) represent a practical situation with a linear inequality in one variable; and
 b) solve one-step linear inequalities in one variable, involving addition or subtraction, and graph the solution on a number line.

Understanding the Standard	Essential Knowledge and Skills
<p>and express simplification. Students should use the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of a, b, or c in this standard):</p> <ul style="list-style-type: none"> - Commutative property of addition: $a + b = b + a$. - Commutative property of multiplication: $a \cdot b = b \cdot a$. - Subtraction and division are neither commutative nor associative. - Identity property of addition (additive identity property): $a + 0 = a$ and $0 + a = a$. - Identity property of multiplication (multiplicative identity property): $a \cdot 1 = a$ and $1 \cdot a = a$. - The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by one is the number. There are no identity elements for subtraction and division. - Inverses are numbers that combine with other numbers and result in identity elements (e.g., $5 + (-5) = 0$; $\frac{1}{5} \cdot 5 = 1$). - Inverse property of addition (additive inverse property): $a + (-a) = 0$ and $(-a) + a = 0$. - Inverse property of multiplication (multiplicative inverse property): $a \cdot \frac{1}{a} = 1$ and $\frac{1}{a} \cdot a = 1$. - Zero has no multiplicative inverse. - Multiplicative property of zero: $a \cdot 0 = 0$ and $0 \cdot a = 0$. - Addition property of inequality: If $a < b$, then $a + c < b + c$; if $a > b$, then $a + c > b + c$ (this property also applies to \leq and \geq). - Subtraction property of inequality: If $a < b$, then $a - c < b - c$; if $a > b$, then $a - c > b - c$ (this property also applies to \leq and \geq). - Substitution property: If $a = b$ then b can be substituted for a in any expression, equation or inequality. 	